

# Short-distance RG-analysis of $X(3872)$ radiative decays

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## Abstract

We present a renormalization-group analysis of the  $X(3872)$  radiative decays into  $J/\psi\gamma$  and  $\psi(2S)\gamma$ . We assume a  $D\bar{D}^*$  molecule for the  $X$  long-distance structure and parametrize its short-distance physics as contact interactions. Using effective field theory techniques and power-divergence subtraction scheme, we find that short- and long-distance physics are equally important in these decays. Our calculations set a lower limit to the corresponding decay widths, which can in principle be tested experimentally. Our results may be used as guide to build models for the  $X$  short-distance.

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## I. INTRODUCTION

The interest in the physics of heavy quarkonia was spurred, more than a decade ago, by the discovery of the exotic meson  $X(3872)$  [1–3], whose physical properties do not fit in the historically well-succeeded quark model. Soon after, a new “particle zoo” of exotic particles was uncovered by several collaborations around the world. The challenge imposed by the so-called  $X$ ,  $Y$ , and  $Z$  states triggered alternative explanations for their structures, such as tetraquarks, molecules, gluonic excitations, mixtures, and others [4–7]. The latest discoveries of charged mesons in the charmonium and bottomonium sectors put in check explanations based on either the conventional quark model, or hybrids of charmonium and gluonic excitations since, besides the  $c\bar{c}$  pair, two additional quarks must exist to provide the charge of these states. These recent findings highlight the complexities of the strong force in its non-perturbative regime, with the promise of improve our understanding about the emergence of confinement in QCD.

Despite the large amount of theoretical investigations in the literature, very little is known about the  $X(3872)$  structure (denoted by  $X$  from now on). However, the purely molecular interpretation is very appealing, since it has a mass remarkably close to the  $D\bar{D}^*$  threshold [8–10] and its small width is not easily accommodated in the conventional quarkonium picture. On the experimental side, only recently the  $X$  quantum numbers were confirmed as  $J^{PC} = 1^{++}$  by the LHCb collaboration [11]. This result practically rules out any conventional charmonium explanation, albeit not enough to distinguish among the remaining exotic possibilities.

Swanson, in Ref. [12], suggested looking at the  $X$  radiative decays into  $\gamma J/\psi$  and  $\gamma\psi(2S)$  as one of the promising tests for its molecular nature. His molecular model calculation includes both neutral and charged  $D\bar{D}^*$  states, decaying to a photon via light quark annihilation, and smaller components of  $\rho J/\psi$  and  $\omega J/\psi$  decaying to a photon via vector-meson dominance. He obtains  $\Gamma(X \rightarrow \gamma J/\psi) = 8$  keV and  $\Gamma(X \rightarrow \gamma\psi') = 0.03$  keV. His quark model calculations, though very sensitive to the detailed assumptions involved, give  $\Gamma(X \rightarrow \gamma J/\psi) \sim 70\text{--}140$  keV and  $\Gamma(X \rightarrow \gamma\psi') \sim 95$  keV. Based on these results, the author claims that radiative decays, especially decaying into the  $\psi(2S)$  channel (here denoted by  $\psi'$ ), would be a sharp test for the  $X$  molecular nature. In 2009 BaBar measured the ratio

$$R \equiv \frac{\Gamma[X \rightarrow \gamma\psi']}{\Gamma[X \rightarrow \gamma J/\psi]} \quad (1)$$

obtaining  $R = 3.4 \pm 1.4$  [13]. Later attempt to measure the same ratio was done by the Belle collaboration. They could not find any signal though, setting only the upper limit  $R < 2.1$  at 90% confidence level [14]. Recently, the LHCb collaboration reported  $R = 2.46 \pm 0.64 \pm 0.29$ , where the first uncertainty is statistical and the second is systematic [15]. The latter concludes that the experimental result does not support the pure molecular picture, favoring charmonium [16–20] or mixtures of molecule and charmonium [21–23]. However this conclusion, which is based on the results of Ref. [12], is disputable, as indicated by further calculations in the molecular approach.

Using phenomenological meson Lagrangians and assuming the  $X$  to be a loosely-bound  $D^0\bar{D}^{*0}$  molecule, Dong and collaborators [24] calculated the radiative decay  $X \rightarrow \gamma J/\psi$ , obtaining an upper limit of 118.9 keV, compatible with some quark model predictions [12]. Their calculation is not very sensitive to variations on the binding energy, but depends on their form-factor parameter  $\Lambda_M$ . They conclude that their decay width is fully compatible with a predominantly molecular nature of  $X$ , allowing a very small admixture of  $c\bar{c}$ . In a latter work [25], the same authors addressed, besides a couple of hadronic decays, radiative decays into both  $J/\psi$  and  $\psi'$  channels. Their values for the ratio  $R$  were compared against the experimental one available at the time, from BaBar [13]. With essentially the same molecular approach used before [24], but with different quark model

approaches for their  $c\bar{c}$  component, they find a subtle interplay between these two components, depending on the  $c\bar{c}$  model and the  $X$  binding energy.

Guo *et al.* [26] investigated the imprints of the long-distance  $D\bar{D}^*$  molecular structure of  $X$  on the radiative decays  $X \rightarrow \gamma J/\psi$  ( $\Gamma_{\gamma J/\psi}$ ) and  $X \rightarrow \gamma \psi'$  ( $\Gamma_{\gamma \psi'}$ ), with an effective field theory approach. Contrary to [12], they conclude that the radiative decays do not allow one to draw conclusions about the nature of the  $X$ . However, their analysis focus only at the long-distance loop contributions to the radiative amplitude, without explicitly considering the short-distance contributions, parametrized as contact-like interactions in the effective Lagrangian. The main purpose of this work is to perform a proper renormalization-group analysis of both loop and contact contributions to these radiative decays, therefore, complementing the studies of Ref. [26].

Nevertheless, we adopt here a different prescription to regularize the loop integrals, the power-divergence subtraction (PDS) scheme [27, 28]. Devised to handle the non-perturbative aspect of the nucleon-nucleon interaction, it soon became an alternative regularization method in other non-perturbative systems such as cold-atoms, exotic mesons, and nuclear clusters [29–33]. Based on dimensional regularization, the method consists of subtractions, beyond the  $D = 4$  dimensions of the usual modified minimal subtraction ( $\overline{\text{MS}}$ ) scheme, at lower dimensions to account for power-law divergences [34]. The latter are required in order to guarantee non-trivial renormalization-group properties characteristic of weakly-bound systems [35, 36], like the scaling limit and the Efimov effect [29]. In fact, for non-perturbative systems the usual  $\overline{\text{MS}}$  scheme seems to fail in reproducing a non-trivial scattering amplitude [36].

## II. RADIATIVE DECAY AMPLITUDES

The interactions are derived from effective Lagrangians based on heavy-meson and chiral symmetries [26, 37–41]. The loop diagrams (a)–(e) from Fig. 1 were calculated in Ref. [26], and for the sake of completeness we reproduce below the relevant expressions.

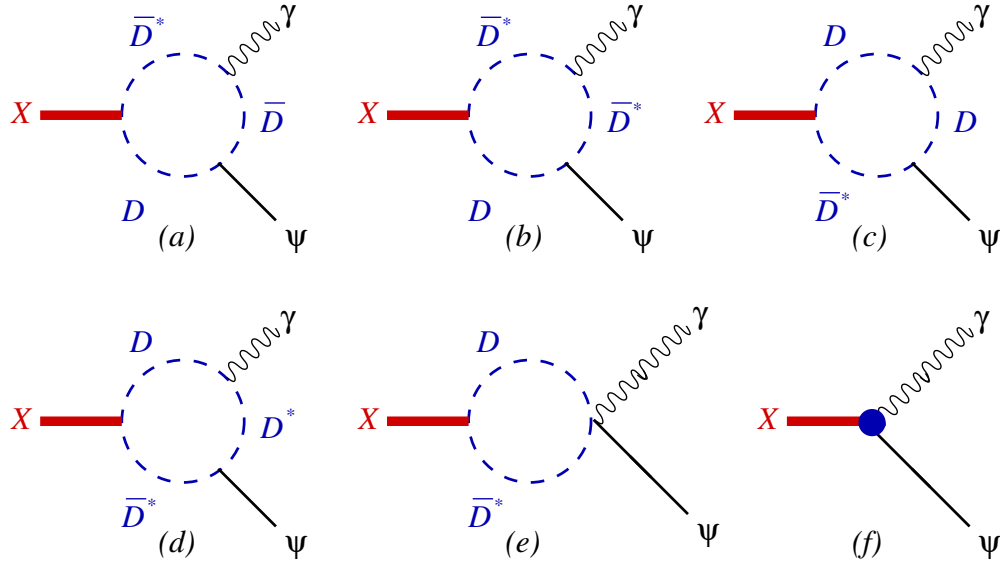


FIG. 1. Diagrams that contribute to the  $X$  radiative decay. See text for details.

One denotes  $\psi$  generically for both  $J/\psi$  and  $\psi'$ . The hidden-charm mesons  $X$  and  $\psi$  have

masses  $M_X$  and  $M_\psi$  while the open-charm mesons  $D^*$  and  $D$  have masses  $m_\star$  and  $m$ , respectively. The  $X$  long-range structure is coupled to a  $D\bar{D}^*$  pair with strength  $x = x_{\text{nr}}\sqrt{M_X m_\star m}$  via the interaction Lagrangian

$$\mathcal{L}_X = \frac{x}{\sqrt{2}} X_\sigma^\dagger \left( \bar{D}^{*\sigma} D + \bar{D} D^{*\sigma} \right) + \text{H.c.}, \quad (2)$$

where the open-charm meson fields stand for both neutral and charged ones [26], and  $x_{\text{nr}}^{(0)} = 0.97_{-0.97}^{+0.40} \text{ GeV}^{-1/2}$  was obtained from a non-relativistic relation with the binding energy [42]. We define  $x_{\text{nr}}^{(0)} \equiv x_{\text{nr}}(\mu_0)$ , the renormalized coupling of  $X$  to the  $D\bar{D}^*$  pair at the scale  $\mu_0 = M_X$ .

The interaction of the open-charm mesons with the hidden-charm  $\psi$  reads

$$\begin{aligned} \mathcal{L}_\psi = i\psi^{\mu\dagger} \Big\{ & g_{\bar{D}D} (\bar{D} \overset{\leftrightarrow}{\partial}_\mu D) - g_{\bar{D}^*D} \epsilon_{\mu\nu\alpha\beta} \left[ (\partial^\alpha \bar{D}^{*\nu}) (\partial^\beta D) - (\partial^\beta \bar{D}) (\partial^\alpha D^{*\nu}) \right] \\ & - g_{\bar{D}^*D^*} \left[ \bar{D}_\nu^* \overset{\leftrightarrow}{\partial}_\mu D^{*\nu} + (\partial_\nu \bar{D}_\mu^*) D^{*\nu} - \bar{D}^{*\nu} (\partial_\nu D_\mu^*) \right] \Big\} + \text{H.c.}, \end{aligned} \quad (3)$$

with the couplings related to a single parameter  $g_2$  via heavy-quark symmetry [26, 40, 41]:

$$\frac{g_{\bar{D}D}}{\sqrt{M_\psi}} \frac{1}{m} = \frac{g_{\bar{D}^*D}}{\sqrt{M_\psi}} \sqrt{\frac{m_\star}{4m}} = \frac{g_{\bar{D}^*D^*}}{\sqrt{M_\psi}} \frac{1}{m_\star} = g_\psi. \quad (4)$$

Interactions with the emitted photon have two distinct origins. Electric interactions have no additional parameters. They contribute to diagrams (b), (c), and (e), via minimal substitution  $\partial_\mu \rightarrow \partial_\mu + ieA_\mu$  in the kinetic term of the  $D^*$ , kinetic term of the  $D$ , and interaction term of the  $\bar{D}^*D\psi$  Lagrangians, respectively. Magnetic interactions are derived from the covariant generalization of the non-relativistic heavy-meson Lagrangian [26, 43, 44],

$$\begin{aligned} \mathcal{L}_m = ie m_\star F_{\mu\nu} D_i^{*\mu\dagger} \left( \beta Q_{ij} - \frac{Q_c}{m_c} \delta_{ij} \right) D_j^{*\nu} \\ + e \sqrt{mm_\star} \epsilon_{\lambda\mu\alpha\beta} v^\alpha \partial^\beta A^\lambda \left[ D_i^{*\mu\dagger} \left( \beta Q_{ij} + \frac{Q_c}{m_c} \delta_{ij} \right) D_j + \text{H.c.} \right], \end{aligned} \quad (5)$$

where  $v^\mu$  is the four-velocity of the heavy quark with  $v^2 = 1$ ,  $Q = \text{diag}(Q_u, Q_d)$  is the charge matrix of the light quarks,  $Q_c = 2/3$  and  $m_c$  are the charmed quark charge and mass, respectively. They contribute to diagrams (a), (b), and (d). The extra parameter  $\beta \sim 1/(380 \text{ MeV})$  takes into account the non-perturbative dynamics of the light quark inside the charmed meson [26].

The loop amplitudes from diagrams (a)-(e) are written as

$$\mathcal{M}^{\text{loop}} = \frac{e}{\sqrt{2}} x_{\text{nr}} g_\psi m \sqrt{M_X M_\psi} \epsilon_\psi^\mu(p') \epsilon_X^\sigma(p) \epsilon_\gamma^\lambda(q) \int \frac{d^4 k}{(2\pi)^4} S^{v\sigma}(k) S(k-p) J_{\mu\nu\lambda}(k), \quad (6)$$

where  $S^{v\sigma}(S)$  is the  $\bar{D}^*$  ( $D$ ) propagator with momentum  $k$  ( $k-p$ ). The polarization vectors  $\epsilon_\psi^\mu$ ,  $\epsilon_X^\sigma$ , and  $\epsilon_\gamma^\lambda$  stand for  $X$ ,  $\psi$ , and  $\gamma$  with external momenta  $p$ ,  $p' = p - q$ , and  $q$ , respectively. The explicit contributions of each diagram to  $J_{\mu\nu\lambda}$  is given by Eqs. (24)-(29) of Ref. [26].

Diagram (f) is the short-distance  $X\gamma\psi$  interaction that renormalizes the ultraviolet divergences present in (6). The corresponding amplitude reads

$$\mathcal{M}^{\text{cont}} = C_\psi \epsilon_\psi^\mu(p') \epsilon_X^\sigma(p) \epsilon_\gamma^\lambda(q) \epsilon_{\mu\sigma\lambda\nu} q^\nu. \quad (7)$$

As pointed out in [26], the necessity of having this term to cancel the divergences of the loop diagrams means that the  $X$  radiative decays are sensitive to both its long- and short-distance structure, making them unsuitable as probes exclusively of the former. Nevertheless, renormalization-group techniques can be used to estimate the strength of the short-range interactions at a limited range of energy scale. This is the main goal of this work. In Ref. [26], the strength of diagram (f) after renormalization,  $C_\psi^{(r)}$ , was set to zero. We take advantage of the most recent experimental value of  $R$  by the LHCb collaboration to obtain the renormalized values of  $C_\psi^{(r)}$  in both  $J/\psi$  and  $\psi'$  channels.

### III. RESULTS AND DISCUSSIONS

The partial width of the  $X$  radiative decay, when the initial and final polarization states are not measured, is given by

$$\Gamma = \frac{M_X^2 - M_\psi^2}{48\pi M_X^3} |\mathcal{M}|^2, \quad (8)$$

where the total amplitude squared reads

$$\begin{aligned} |\mathcal{M}|^2 &= \sum_{\text{all pols.}} \mathcal{M}_{\mu'\sigma'\lambda'} \mathcal{M}_{\mu\sigma\lambda}^* \left( \epsilon_{(X)}^{\sigma'}(p) \epsilon_{(X)}^{*\sigma}(p) \right) \left( \epsilon_{(\psi)}^{\mu'}(p') \epsilon_{(\psi)}^{*\mu}(p') \right) \left( \epsilon_{(\gamma)}^{\lambda'}(q) \epsilon_{(\gamma)}^{*\lambda}(q) \right) \\ &= \mathcal{M}_{\mu'\sigma'\lambda'} \mathcal{M}_{\mu\sigma\lambda}^* \left( \frac{p^{\sigma'} p^\sigma}{M_X^2} - g^{\sigma'\sigma} \right) \left( \frac{p'^{\mu'} p'^\mu}{M_\psi^2} - g^{\mu'\mu} \right) \left( -g^{\lambda'\lambda} \right). \end{aligned} \quad (9)$$

We made use of the Mathematica software to deal with contractions of the Lorentz indices and algebraic manipulations. The loop integrals in Eq. (6) were handled with the usual Feynman parametrizations, with the remaining integrations solved numerically with a Gauss-Legendre quadrature.

In order to estimate qualitatively the importance of short-distance physics, we first compute the ratio  $R$  from Eq. (1) considering only the long-range loop diagrams (a)-(e), which we shall denote by  $R^{\text{loop}}$ . At this point the analysis is similar to the one from Ref. [26]. It is evident from Eq. (6) that, in this case, the dependence on  $x_{\text{nr}}$  is cancelled in the ratio  $R^{\text{loop}}$ . However,  $R^{\text{loop}}$  remains very sensitive to the ratio  $(g_{\psi'}/g_{J/\psi})^2$ , which is poorly known. From the leptonic decay widths of  $J/\psi$  and  $\psi'$ , Ref. [25] obtains  $g_{\psi'}/g_{J/\psi} \sim 1.67$ , which is the central value that we adopt. We allow a variation around the natural band  $1 \lesssim g_{\psi'}/g_{J/\psi} \lesssim 2.5$ , which should account for uncertainties in both phenomenological extraction as well as renormalization evolution of this ratio.

Fig. 2 shows  $R^{\text{loop}}$  as a function of the renormalization scale  $\mu$ . We choose the interval  $3 \text{ GeV} \leq \mu \leq 5 \text{ GeV}$ , not too far from  $\mu = M_X$ , the relevant scale of the problem. The dashed (blue) curve essentially reproduces the results of Ref. [26]. The solid (red) curve is our result, with the loop integrals regularized within the PDS scheme. The (gray) horizontal band is the LHCb experimental value [15], with uncertainties added in quadrature. Though these results are quite different from each other and not yet properly renormalized, one is still able to refute the conclusions from Ref. [12, 15], that a ratio  $R$  much larger than  $\sim 10^{-3}$  would disfavor a molecular nature of  $X$ . Regarding the behavior of these two different curves, a few comments are in order. First, since  $R$  is an observable, it should not depend on  $\mu$ . Therefore, Fig. 2 indicates the need of proper renormalization. Second, we draw attention to the logarithmic scale of the figure. In the PDS scheme adopted here, the dependence of  $R^{\text{loop}}$  on the renormalization scale  $\mu$  is much stronger

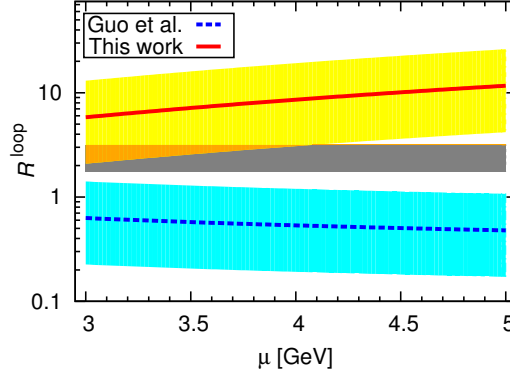


FIG. 2. Ratio  $R^{\text{loop}}$  of the branching fractions defined by Eq. (1). See text for details.

than in [26], which uses the standard  $\overline{\text{MS}}$ -scheme. This is somehow expected, since PDS-regulated loops take into account power divergences that are set to zero in  $\overline{\text{MS}}$ . Power divergences lead to a richer structure in the renormalization-group (RG) evolution path, allowing the existence of a non-trivial fixed point [35] that describes the non-perturbative aspects of weakly-bound systems. Such RG constraints can induce a larger dependence of the short-distance contact couplings  $C_\psi$ 's on  $\mu$ , as shown in the following. Most of power divergences come from magnetic interactions, c.f. Eqs. (24)–(28) of Ref. [26], meaning that they are more sensitive to short-distance physics. Note that there are a few examples in weakly-bound nuclear systems where the short-distance sensitivity of magnetic interactions is also observed—see, for instance, Refs. [32, 33].

The previous discussion points to the need of including explicitly the contact interactions from diagram (f) and perform a proper RG analysis. For practical reasons, we find more convenient to impose the RG-constraint on the decay width, that is,

$$\frac{\partial \Gamma}{\partial \mu} = 0. \quad (10)$$

This condition is imposed, numerically, on each decay channel  $\gamma J/\psi$  and  $\gamma \psi'$ , tied to the experimental constraint  $R \approx 2.46$  [15]. Since both  $\Gamma_{\gamma \psi'}$  and  $\Gamma_{\gamma J/\psi}$ , contrary to their ratio, are not well-determined, we choose a few representative values of  $\Gamma_{\gamma J/\psi}$  as initial boundary condition in our RG-equation (10). One finds two sets of solutions for the  $\mu$ -dependent contact terms,  $C_{J/\psi}$  and  $C_{\psi'}$ , which reflects the fact that we impose the RG-constraint essentially on the modulus-squared of the decay amplitude. We present only one of these sets, since the other leads to decay widths of the order of tenths of MeV while the total decay width of the  $X$  has an upper limit of 1.2 MeV [45]. We assume that all  $\mu$ -dependence is given by the couplings  $C_\psi$ 's, ignoring eventual  $\mu$ -dependences on the couplings  $x_{\text{nr}}$  and  $g_\psi$ .

In figure 3 we present our RG results for the short-distance couplings  $C_{J/\psi}$  and  $C_{\psi'}$ . As indicated, the curves correspond to different values of  $\Gamma_{\gamma J/\psi}$ . Within the selected ranges of  $\mu$  and  $\Gamma_{\gamma J/\psi}$ , one notices a smooth variation on  $C_{J/\psi}$ , within  $\approx 30\%$ , about the same relative error on  $R$  reported by LHCb. We stress that the extraction of the couplings  $C_\psi$ 's depend on  $x_{\text{nr}}$  via Eq. (6) and has a much larger relative theoretical uncertainty. On the other hand,  $C_{\psi'}$  exhibits a stronger variation with  $\mu$ , meaning that  $\Gamma_{\gamma \psi'}$  is more sensitive to the short-distance physics not dynamically taken into account by our effective theory. It is important to emphasize that, from the EFT point of view, short-distance physics means not only compact configurations like charmonium or tetraquark, but also heavier molecular states that are integrated-out from the effective theory, for instance, a virtual

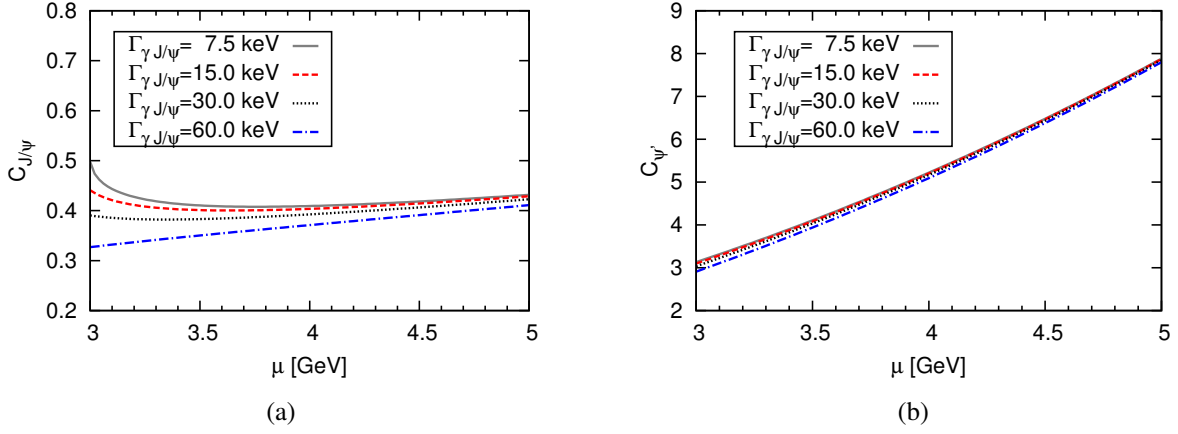


FIG. 3. Short-distance contact interactions  $C_{J/\psi}$  and  $C_{\psi'}$ , as functions of the renormalization scale  $\mu$ .

$D_s \bar{D}_s^*$  contribution.

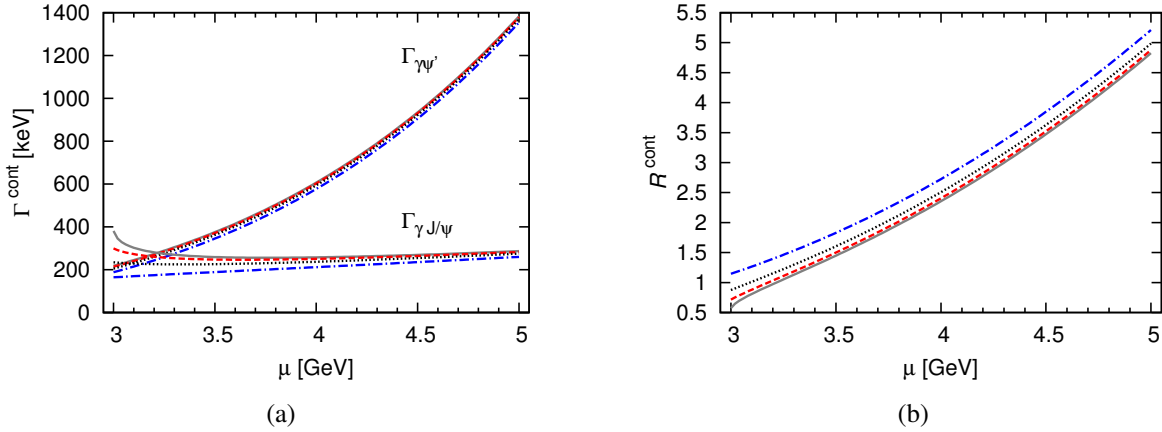


FIG. 4. Contributions of the short-range interactions to the decay widths (a) and the ratio  $R$  (b), as functions of the renormalization scale  $\mu$ . The curves follow the same labels from Fig. 3.

To estimate the relevance of the short-distance interactions to  $X$  radiative decays, we show in Fig. 4 only their contributions to the decay widths  $\Gamma_{\gamma J/\psi}$  and  $\Gamma_{\gamma \psi'}$ , as well as the ratio  $R$ . To qualitatively interpret these results, let us focus on the dash-dotted (blue) curve of  $\Gamma_{\gamma J/\psi}$ . This has an input value of 60 keV. However, Fig. 4(a) shows that its short-range interactions contribute about 200 keV. That implies a large cancellation between the long- ( $\mathcal{M}^{\text{loop}}$ ) and short- ( $\mathcal{M}^{\text{cont}}$ ) distance terms to generate the smaller width of 60 keV. In other words, the interplay between  $\mathcal{M}^{\text{loop}}$  and  $\mathcal{M}^{\text{cont}}$  is of a delicate balance, both having the same importance to the decay. This is even more dramatic in the case of  $\Gamma_{\gamma \psi'}$ . Such large cancellations between long- and short-distance terms may be a consequence of an underlying symmetry and is a question worth pursuing. We also checked that this qualitative cancellation holds for the  $\overline{\text{MS}}$ -regulated loops, as much as for  $\Gamma_{\gamma J/\psi}$  but less dramatic for  $\Gamma_{\gamma \psi'}$ . The  $\mu$ -dependence shown in figure 4 may also be relevant in guiding theoretical models for the short-distance part.

In EFT approach, representations of short-range physics as contact interactions mean that all dynamical effects not explicitly taken into account may be relevant only beyond the EFT

scale. That includes the opening of high momenta thresholds, leading to imaginary terms in the amplitude. Therefore, the constraint that the short-range couplings  $C_\psi$  remain real-valued within the energy range of the effective theory assures that there are no opening of high-energy thresholds. However, one notices numerically the appearance of an imaginary part on  $C_\psi$  in the evolution of our RG-equation (10), depending on the initial conditions. Choosing the range  $3 \text{ GeV} \lesssim \mu \lesssim 5 \text{ GeV}$ , to keep  $C_\psi$  real one finds the restrictions

$$\Gamma_{\gamma J/\psi} \gtrsim 7.5 \text{ keV} \quad \text{and} \quad \Gamma_{\gamma \psi'} \gtrsim 18.5 \text{ keV}. \quad (11)$$

Though the precise values depend on the EFT range, these numbers can be put to experimental scrutiny.

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- [1] S.-K. Choi *et al.* (Belle Collaboration), Phys. Rev. Lett. **91** (2003), 10.1103/PhysRevLett.91.262001.
  - [2] D. Acosta *et al.* (CDF Collaboration), Phys. Rev. Lett. **93**, 072001 (2004).
  - [3] V. M. Abazov *et al.* (D0 Collaboration), Phys. Rev. Lett. **93**, 162002 (2004).
  - [4] G. Bauer, Int. J. Mod. Phys. **A21**, 959 (2006).
  - [5] E. S. Swanson, Phys. Rept. **429**, 243 (2006).
  - [6] M. Nielsen, F. S. Navarra, and S. H. Lee, Phys. Rept. **497**, 41 (2010).
  - [7] N. Brambilla *et al.*, Eur. Phys. J. **C71**, 1534 (2011).
  - [8] M. B. Voloshin and L. B. Okun, JETP Lett. **23**, 333 (1976), [Pisma Zh. Eksp. Teor. Fiz.23,369(1976)].
  - [9] A. De Rujula, H. Georgi, and S. L. Glashow, Phys. Rev. Lett. **38**, 317 (1977).
  - [10] N. A. Tornqvist, Phys. Lett. **B590**, 209 (2004), arXiv:hep-ph/0402237 [hep-ph].
  - [11] R. Aaij *et al.*, Phys. Rev. Lett. **110**, 222001 (2013).
  - [12] E. S. Swanson, Phys. Lett. B **598**, 197 (2004).
  - [13] B. Aubert *et al.* (BaBar Collaboration), Phys. Rev. Lett. **102**, 132001 (2009).
  - [14] V. Bhardwaj *et al.* (Belle Collaboration), Phys. Rev. Lett. **107**, 091803 (2011).
  - [15] R. Aaij *et al.* (LHCb), Nucl. Phys. **B886**, 665 (2014), arXiv:1404.0275 [hep-ex].
  - [16] T.-H. Wang and G.-L. Wang, Phys. Lett. B **697**, 233 (2011).
  - [17] T. Barnes and S. Godfrey, Phys. Rev. D **69**, 054008 (2004).
  - [18] T. Barnes, S. Godfrey, and E. S. Swanson, Phys. Rev. D **72**, 054026 (2005).
  - [19] A. M. Badalian, V. D. Orlovsky, Y. A. Simonov, and B. L. G. Bakker, Phys. Rev. D **85**, 114002 (2012).
  - [20] A. M. Badalian, Y. A. Simonov, and B. L. G. Bakker, Phys. Rev. D **91**, 056001 (2015).
  - [21] M. Nielsen and C. M. Zanetti, Phys. Rev. D **82**, 116002 (2010).
  - [22] C. M. Zanetti and M. Nielsen, Nucl. Phys. B **207**, 253 (2010).
  - [23] M. Takizawa, S. Takeuchi, and K. Shimizu, Few-Body Systems **55**, 779 (2014).
  - [24] Y. Dong, A. Faessler, T. Gutsche, and V. E. Lyubovitskij, Phys. Rev. D **79**, 094013 (2008).
  - [25] Y. Dong, A. Faessler, T. Gutsche, and V. E. Lyubovitskij, J. Phys. **G38**, 015001 (2011), arXiv:0909.0380 [hep-ph].
  - [26] F.-K. Guo, C. Hanhart, Y. Kalashnikova, U.-G. Meißner, and A. Nefediev, Phys. Lett. B **742**, 394 (2015).



- [27] D. B. Kaplan, M. J. Savage, and M. B. Wise, Phys. Lett. **B424**, 390 (1998), arXiv:nucl-th/9801034 [nucl-th].
- [28] D. B. Kaplan, M. J. Savage, and M. B. Wise, Nucl. Phys. **B534**, 329 (1998), arXiv:nucl-th/9802075 [nucl-th].
- [29] E. Braaten and H. W. Hammer, Phys. Rept. **428**, 259 (2006), arXiv:cond-mat/0410417 [cond-mat].
- [30] E. Braaten and M. Kusunoki, Phys. Rev. **D72**, 014012 (2005), arXiv:hep-ph/0506087 [hep-ph].
- [31] G. Rupak and R. Higa, Phys. Rev. Lett. **106**, 222501 (2011), arXiv:1101.0207 [nucl-th].
- [32] L. Fernando, R. Higa, and G. Rupak, Eur. Phys. J. **A48**, 24 (2012), arXiv:1109.1876 [nucl-th].
- [33] L. Fernando, A. Vaghani, and G. Rupak, (2015), arXiv:1511.04054 [nucl-th].
- [34] D. R. Phillips, S. R. Beane, and M. C. Birse, J. Phys. **A32**, 3397 (1999), arXiv:hep-th/9810049 [hep-th].
- [35] M. C. Birse, J. A. McGovern, and K. G. Richardson, Phys. Lett. **B464**, 169 (1999), arXiv:hep-ph/9807302 [hep-ph].
- [36] D. R. Phillips, S. R. Beane, and T. D. Cohen, Annals Phys. **263**, 255 (1998), arXiv:hep-th/9706070 [hep-th].
- [37] G. Burdman and J. F. Donoghue, Phys. Lett. B **280**, 287 (1992).
- [38] M. T. AlFiky, F. Gabbiani, and A. A. Petrov, Phys. Lett. B **640**, 238 (2006).
- [39] S. Fleming, M. Kusunoki, T. Mehen, and U. Van Kolck, Phys. Rev. D **76**, 1 (2007).
- [40] F.-K. Guo, C. Hanhart, G. Li, U.-G. Meißner, and Q. Zhao, Phys. Rev. D **83**, 034013 (2011).
- [41] P. Colangelo, F. De Fazio, and T. N. Pham, Phys. Rev. **D69**, 054023 (2004), arXiv:hep-ph/0310084 [hep-ph].
- [42] F.-K. Guo, C. Hanhart, U.-G. Meiner, Q. Wang, and Q. Zhao, Phys. Lett. **B725**, 127 (2013), arXiv:1306.3096 [hep-ph].
- [43] J. F. Amundson, C. G. Boyd, E. E. Jenkins, M. E. Luke, A. V. Manohar, J. L. Rosner, M. J. Savage, and M. B. Wise, Phys. Lett. **B296**, 415 (1992), arXiv:hep-ph/9209241 [hep-ph].
- [44] H.-Y. Cheng, C.-Y. Cheung, G.-L. Lin, Y. C. Lin, T.-M. Yan, and H.-L. Yu, Phys. Rev. **D47**, 1030 (1993), arXiv:hep-ph/9209262 [hep-ph].
- [45] K. A. Olive *et al.* (Particle Data Group), Chin. Phys. **C38**, 090001 (2014).